**Algorithm**

par1 = rnorm(18)

## The parameters for the second segment

par2 = rnorm(18)

## The parameter for the likelihood that a randomly chosen individual belongs to segment 1

p1 = rnorm(1)

## The combined parameters are

par\_2 = t(cbind(t(par1),t(par2),t(p1)))

N = 1918

ll\_2 <- function(beta)

{

res = 0

## To calculate the likelohood for the first segment

M11 = as.matrix(train\_set)[,1:18] %\*% beta[1:18]

M12 = as.matrix(train\_set)[,19:36] %\*% beta[1:18]

M13 = as.matrix(train\_set)[,37:54] %\*% beta[1:18]

M1 = cbind(exp(M11),exp(M12),exp(M13))

## The likelihood for each of the three alternatives in the first segment

M1 = M1 / rowSums(M1)

M21 = as.matrix(train\_set)[,1:18] %\*% beta[19:36]

M22 = as.matrix(train\_set)[,19:36] %\*% beta[19:36]

M23 = as.matrix(train\_set)[,37:54] %\*% beta[19:36]

M2 = cbind(exp(M21),exp(M22),exp(M23))

## The likelihood for each of the three alternatives in the first segment

M2 = M2 / rowSums(M2)

M\_2 = beta[37] \* M1 + (1 - beta[37]) \* M2

MP\_2 = M\_2[cbind(seq(1,length(train\_choice)),train\_choice)]

res = res + sum(-log(MP\_2))

return (res)

}

ML\_2 = nlm(ll\_2,par\_2,hessian = TRUE)

Results:

Estimate SE Tvalue minusll

[1,] 0.47 0.18 2.60 3582.5

[2,] 1.07 0.18 5.79 3582.5

[3,] 0.36 0.06 5.68 3582.5

[4,] 0.08 0.08 1.07 3582.5

[5,] 0.22 0.13 1.67 3582.5

[6,] 0.21 0.11 1.90 3582.5

[7,] 0.39 NaN NaN 3582.5

[8,] 0.41 0.10 3.97 3582.5

[9,] 0.51 0.14 3.73 3582.5

[10,] 0.89 0.13 7.04 3582.5

[11,] 0.46 0.10 4.43 3582.5

[12,] 0.65 0.07 8.65 3582.5

[13,] 0.01 0.11 0.11 3582.5

[14,] 0.03 0.10 0.31 3582.5

[15,] -0.44 0.11 -4.02 3582.5

[16,] -0.94 NaN NaN 3582.5

[17,] -1.34 0.11 -12.11 3582.5

[18,] -3.03 NaN NaN 3582.5

[19,] -1.31 NaN NaN 3582.5

[20,] 0.68 0.40 1.69 3582.5

[21,] 0.38 NaN NaN 3582.5

[22,] 0.27 NaN NaN 3582.5

[23,] 0.22 0.63 0.35 3582.5

[24,] 1.89 0.54 3.49 3582.5

[25,] 0.41 NaN NaN 3582.5

[26,] -0.36 NaN NaN 3582.5

[27,] 1.52 NaN NaN 3582.5

[28,] -0.72 NaN NaN 3582.5

[29,] 0.57 0.21 2.68 3582.5

[30,] 1.67 0.21 7.96 3582.5

[31,] 0.68 NaN NaN 3582.5

[32,] 0.90 0.51 1.77 3582.5

[33,] 0.66 0.42 1.57 3582.5

[34,] 0.05 NaN NaN 3582.5

[35,] -2.05 NaN NaN 3582.5

[36,] 1.43 0.96 1.49 3582.5

[37,] 0.78 NaN NaN 3582.5

Questions: when solving Hessian Matrix, there were many NaNs. Is it implying that we should seek locally optimal solutions rather then global ones? From time to time, this algorithm does not converge, which corresponds to the phenomenon that estimated pi is out of the range of (0,1).

The First Trial : starting point out of range

round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] 0.21 0.10 2.00 Inf

[2,] -1.10 0.10 -11.28 Inf

[3,] 0.54 0.10 5.61 Inf

[4,] -1.53 0.10 -15.55 Inf

[5,] 0.94 0.09 10.96 Inf

[6,] 0.42 0.08 5.03 Inf

[7,] -1.63 0.09 -18.65 Inf

[8,] 1.52 0.09 17.53 Inf

[9,] -0.26 0.08 -3.13 Inf

[10,] 0.15 0.09 1.70 Inf

[11,] -0.30 0.07 -4.33 Inf

[12,] 1.03 0.07 15.40 Inf

[13,] -0.66 0.07 -9.81 Inf

[14,] -0.26 0.07 -3.90 Inf

[15,] -1.47 0.09 -16.85 Inf

[16,] 0.73 0.10 7.72 Inf

[17,] 1.47 0.10 14.16 Inf

[18,] 1.42 0.11 13.01 Inf

[19,] -0.50 0.10 -4.80 Inf

[20,] 1.62 0.10 16.67 Inf

[21,] -0.52 0.10 -5.39 Inf

[22,] -1.02 0.10 -10.42 Inf

[23,] 0.62 0.09 7.19 Inf

[24,] -1.10 0.08 -13.08 Inf

[25,] -0.88 0.09 -10.05 Inf

[26,] 1.65 0.09 19.04 Inf

[27,] -0.20 0.08 -2.42 Inf

[28,] 0.33 0.09 3.65 Inf

[29,] 1.04 0.07 14.86 Inf

[30,] -0.18 0.07 -2.68 Inf

[31,] -0.32 0.07 -4.74 Inf

[32,] 0.35 0.07 5.17 Inf

[33,] -0.09 0.09 -1.07 Inf

[34,] 0.28 0.10 2.90 Inf

[35,] 0.46 0.10 4.46 Inf

[36,] -0.61 0.11 -5.57 Inf

[37,] -0.29 0.10 -2.80 Inf

>

> ## The estimated probability for the first segment

> ML\_2$estimate[37]

[1] -0.2913582

> performance\_2

[1] 0.3649635

> round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] 2.10 80.83 0.03 251.04

[2,] -4.11 58.19 -0.07 251.04

[3,] 1.41 76.21 0.02 251.04

[4,] 0.19 69.77 0.00 251.04

[5,] 1.95 41.54 0.05 251.04

[6,] -0.37 17.07 -0.02 251.04

[7,] 2.33 NaN NaN 251.04

[8,] -1.98 55.44 -0.04 251.04

[9,] -1.08 36.59 -0.03 251.04

[10,] -0.03 18.53 0.00 251.04

[11,] -0.80 NaN NaN 251.04

[12,] 1.59 13.40 0.12 251.04

[13,] 2.04 22.28 0.09 251.04

[14,] 1.48 9.55 0.15 251.04

[15,] 0.63 10.76 0.06 251.04

[16,] -3.40 4.67 -0.73 251.04

[17,] -1.84 10.82 -0.17 251.04

[18,] -3.55 20.10 -0.18 251.04

[19,] -8.20 276.41 -0.03 251.04

[20,] 4.78 109.73 0.04 251.04

[21,] -1.04 134.64 -0.01 251.04

[22,] 2.87 356.38 0.01 251.04

[23,] 1.74 323.70 0.01 251.04

[24,] 1.57 192.77 0.01 251.04

[25,] 1.41 359.11 0.00 251.04

[26,] 1.46 165.81 0.01 251.04

[27,] 3.92 NaN NaN 251.04

[28,] 4.30 146.52 0.03 251.04

[29,] -3.24 203.55 -0.02 251.04

[30,] -2.59 157.59 -0.02 251.04

[31,] -0.78 120.26 -0.01 251.04

[32,] 0.71 132.46 0.01 251.04

[33,] 4.93 257.01 0.02 251.04

[34,] -5.32 NaN NaN 251.04

[35,] -0.83 285.80 0.00 251.04

[36,] 1.40 339.66 0.00 251.04

[37,] 0.58 NaN NaN 251.04

> ML\_2$estimate[37]

[1] 0.5809365

> performance\_2

[1] 0.3868613

> round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] 0.34 0.52 0.65 3569.56

[2,] 0.93 0.41 2.28 3569.56

[3,] 0.28 0.77 0.37 3569.56

[4,] -0.57 1.03 -0.55 3569.56

[5,] 0.46 0.28 1.69 3569.56

[6,] -0.15 0.87 -0.17 3569.56

[7,] 0.21 0.63 0.34 3569.56

[8,] 1.10 0.56 1.95 3569.56

[9,] 1.71 0.77 2.22 3569.56

[10,] 1.95 1.35 1.44 3569.56

[11,] -0.27 0.18 -1.54 3569.56

[12,] -1.26 0.44 -2.85 3569.56

[13,] 0.18 0.48 0.38 3569.56

[14,] 0.32 0.77 0.41 3569.56

[15,] -0.35 0.41 -0.85 3569.56

[16,] -2.02 2.05 -0.98 3569.56

[17,] -1.74 0.65 -2.70 3569.56

[18,] -2.88 1.21 -2.38 3569.56

[19,] 0.21 0.31 0.69 3569.56

[20,] 1.31 0.13 9.78 3569.56

[21,] 0.48 0.34 1.42 3569.56

[22,] 0.57 0.15 3.82 3569.56

[23,] 0.05 0.19 0.27 3569.56

[24,] 0.81 0.15 5.34 3569.56

[25,] 0.34 0.25 1.33 3569.56

[26,] 0.00 0.29 -0.01 3569.56

[27,] 0.24 0.32 0.75 3569.56

[28,] 0.10 0.11 0.88 3569.56

[29,] 0.80 0.48 1.69 3569.56

[30,] 1.59 0.76 2.09 3569.56

[31,] 0.15 0.17 0.88 3569.56

[32,] 0.10 0.25 0.38 3569.56

[33,] -0.44 0.27 -1.59 3569.56

[34,] -0.36 0.35 -1.04 3569.56

[35,] -1.42 0.33 -4.27 3569.56

[36,] -1.72 0.28 -6.09 3569.56

[37,] 0.36 0.16 2.22 3569.56

ML\_2$estimate[37]

[1] 0.3583837

> performance\_2

[1] 0.4014599

> round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] -0.48 NaN NaN 3599.06

[2,] -0.41 1.79 -0.23 3599.06

[3,] -0.25 NaN NaN 3599.06

[4,] 0.33 0.88 0.37 3599.06

[5,] 0.77 NaN NaN 3599.06

[6,] 0.96 NaN NaN 3599.06

[7,] 0.08 0.72 0.10 3599.06

[8,] 0.40 0.87 0.47 3599.06

[9,] -0.25 0.74 -0.34 3599.06

[10,] 1.27 1.88 0.68 3599.06

[11,] -2.38 2.18 -1.09 3599.06

[12,] 1.16 0.91 1.28 3599.06

[13,] -0.22 0.20 -1.12 3599.06

[14,] 0.46 0.59 0.78 3599.06

[15,] 0.76 0.39 1.98 3599.06

[16,] -1.50 1.07 -1.40 3599.06

[17,] -0.73 NaN NaN 3599.06

[18,] -1.46 NaN NaN 3599.06

[19,] 0.17 0.09 1.82 3599.06

[20,] 0.88 0.12 7.21 3599.06

[21,] 0.31 0.08 3.99 3599.06

[22,] 0.16 0.12 1.40 3599.06

[23,] 0.20 0.08 2.71 3599.06

[24,] 0.45 0.05 8.52 3599.06

[25,] 0.27 0.10 2.65 3599.06

[26,] 0.21 0.09 2.36 3599.06

[27,] 0.49 0.10 5.00 3599.06

[28,] 0.62 0.17 3.78 3599.06

[29,] 0.19 0.10 1.98 3599.06

[30,] 0.69 0.14 4.99 3599.06

[31,] 0.11 0.07 1.58 3599.06

[32,] 0.17 0.09 1.88 3599.06

[33,] -0.23 0.09 -2.65 3599.06

[34,] -0.73 0.09 -7.78 3599.06

[35,] -1.16 0.12 -9.63 3599.06

[36,] -1.63 0.11 -15.41 3599.06

[37,] -0.14 0.05 -2.53 3599.06

>

> ## The estimated probability for the first segment

> ML\_2$estimate[37]

[1] -0.137287

> performance\_2

[1] 0.3430657

round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] 0.81 NaN NaN 3535.68

[2,] 0.43 0.17 2.58 3535.68

[3,] 0.64 NaN NaN 3535.68

[4,] -0.11 NaN NaN 3535.68

[5,] 0.00 NaN NaN 3535.68

[6,] 0.07 NaN NaN 3535.68

[7,] 0.47 NaN NaN 3535.68

[8,] 0.88 NaN NaN 3535.68

[9,] 0.97 NaN NaN 3535.68

[10,] 1.80 NaN NaN 3535.68

[11,] 0.68 NaN NaN 3535.68

[12,] 0.80 NaN NaN 3535.68

[13,] 0.06 NaN NaN 3535.68

[14,] 0.24 NaN NaN 3535.68

[15,] -0.65 0.17 -3.88 3535.68

[16,] -1.48 NaN NaN 3535.68

[17,] -2.55 NaN NaN 3535.68

[18,] -4.94 NaN NaN 3535.68

[19,] -0.32 NaN NaN 3535.68

[20,] 2.02 NaN NaN 3535.68

[21,] 0.36 NaN NaN 3535.68

[22,] 0.52 0.25 2.12 3535.68

[23,] 0.58 0.10 5.75 3535.68

[24,] 1.24 NaN NaN 3535.68

[25,] 0.43 NaN NaN 3535.68

[26,] -0.24 0.19 -1.31 3535.68

[27,] 0.51 NaN NaN 3535.68

[28,] -0.38 NaN NaN 3535.68

[29,] 0.16 0.21 0.77 3535.68

[30,] 0.90 NaN NaN 3535.68

[31,] 0.30 0.21 1.40 3535.68

[32,] 0.21 NaN NaN 3535.68

[33,] 0.66 NaN NaN 3535.68

[34,] 0.43 NaN NaN 3535.68

[35,] 0.35 NaN NaN 3535.68

[36,] 0.58 NaN NaN 3535.68

[37,] 0.62 NaN NaN 3535.68

>

> ## The estimated probability for the first segment

> ML\_2$estimate[37]

[1] 0.6170752

> performance\_2

[1] 0.4452555

round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] -0.54 0.41 -1.30 3545.41

[2,] 2.73 NaN NaN 3545.41

[3,] 0.39 0.40 0.96 3545.41

[4,] 0.16 0.37 0.42 3545.41

[5,] -0.02 0.32 -0.05 3545.41

[6,] 1.11 0.33 3.37 3545.41

[7,] 0.35 0.42 0.82 3545.41

[8,] -0.41 0.36 -1.14 3545.41

[9,] 0.60 0.27 2.19 3545.41

[10,] -0.42 0.46 -0.93 3545.41

[11,] 0.13 0.33 0.40 3545.41

[12,] 0.94 0.27 3.43 3545.41

[13,] -0.18 0.33 -0.54 3545.41

[14,] -0.03 0.31 -0.10 3545.41

[15,] -0.18 0.63 -0.29 3545.41

[16,] -0.26 0.32 -0.79 3545.41

[17,] -0.05 0.48 -0.10 3545.41

[18,] 0.28 0.38 0.75 3545.41

[19,] 0.72 0.19 3.87 3545.41

[20,] 0.32 0.24 1.36 3545.41

[21,] 0.44 0.20 2.17 3545.41

[22,] 0.21 0.21 1.02 3545.41

[23,] 0.24 0.17 1.40 3545.41

[24,] 0.29 0.19 1.48 3545.41

[25,] 0.46 0.21 2.22 3545.41

[26,] 0.80 0.19 4.25 3545.41

[27,] 0.93 0.19 4.98 3545.41

[28,] 1.56 0.22 7.09 3545.41

[29,] 0.60 0.14 4.40 3545.41

[30,] 0.78 0.16 4.79 3545.41

[31,] 0.22 0.14 1.55 3545.41

[32,] 0.30 0.15 2.02 3545.41

[33,] -0.32 0.17 -1.88 3545.41

[34,] -1.20 0.06 -20.72 3545.41

[35,] -2.13 0.14 -15.14 3545.41

[36,] -3.71 0.23 -16.32 3545.41

[37,] 0.33 NaN NaN 3545.41

>

> ## The estimated probability for the first segment

> ML\_2$estimate[37]

[1] 0.3323268

performance\_2

[1] 0.3284672

> After Transfering the 37th parameter.

round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] 28.75 NaN NaN 228.63

[2,] 6.40 NaN NaN 228.63

[3,] 31.12 NaN NaN 228.63

[4,] 40.89 NaN NaN 228.63

[5,] 11.68 NaN NaN 228.63

[6,] -55.71 NaN NaN 228.63

[7,] -11.85 NaN NaN 228.63

[8,] -5.53 NaN NaN 228.63

[9,] -26.29 NaN NaN 228.63

[10,] -19.44 NaN NaN 228.63

[11,] 12.45 NaN NaN 228.63

[12,] 16.75 NaN NaN 228.63

[13,] 2.70 NaN NaN 228.63

[14,] 23.41 NaN NaN 228.63

[15,] -21.07 NaN NaN 228.63

[16,] -0.72 NaN NaN 228.63

[17,] -61.04 NaN NaN 228.63

[18,] -39.58 NaN NaN 228.63

[19,] -2.99 NaN NaN 228.63

[20,] 41.25 NaN NaN 228.63

[21,] -3.12 NaN NaN 228.63

[22,] -21.99 NaN NaN 228.63

[23,] -19.68 NaN NaN 228.63

[24,] 28.37 NaN NaN 228.63

[25,] 16.65 200.08 0.08 228.63

[26,] -3.12 NaN NaN 228.63

[27,] 46.58 NaN NaN 228.63

[28,] 53.91 NaN NaN 228.63

[29,] 33.23 NaN NaN 228.63

[30,] 23.26 NaN NaN 228.63

[31,] 5.89 NaN NaN 228.63

[32,] -0.17 NaN NaN 228.63

[33,] -22.14 NaN NaN 228.63

[34,] 5.76 NaN NaN 228.63

[35,] -11.57 NaN NaN 228.63

[36,] -2.97 NaN NaN 228.63

[37,] 0.31 0.21 1.48 228.63

> ML\_2$estimate[37]

[1] 0.3102278

> performance\_2

[1] 0.3649635

> round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] -24.34 0.10 -231.96 230.79

[2,] 44.77 0.10 459.84 230.79

[3,] 1.67 0.10 17.51 230.79

[4,] -44.08 0.10 -450.38 230.79

[5,] -27.23 0.09 -313.61 230.79

[6,] 3.63 0.08 42.76 230.79

[7,] 35.75 0.09 406.29 230.79

[8,] -58.52 0.09 -676.94 230.79

[9,] 2.10 0.08 24.94 230.79

[10,] 15.18 0.09 168.59 230.79

[11,] -16.44 0.07 -234.65 230.79

[12,] -2.86 0.07 -42.61 230.79

[13,] 20.24 0.07 298.38 230.79

[14,] 41.35 0.07 613.15 230.79

[15,] -97.84 0.09 -1127.89 230.79

[16,] 62.34 0.09 662.76 230.79

[17,] -21.21 0.10 -202.56 230.79

[18,] 15.15 0.11 137.69 230.79

[19,] 17.86 0.10 170.28 230.79

[20,] -33.13 0.10 -340.26 230.79

[21,] -25.14 0.10 -263.67 230.79

[22,] -18.13 0.10 -185.29 230.79

[23,] -55.43 0.09 -638.45 230.79

[24,] 11.28 0.08 132.78 230.79

[25,] 3.88 0.09 44.12 230.79

[26,] -29.48 0.09 -340.95 230.79

[27,] 12.19 0.08 145.05 230.79

[28,] 22.23 0.09 246.86 230.79

[29,] 56.81 0.07 811.04 230.79

[30,] 21.87 0.07 325.62 230.79

[31,] 17.24 0.07 254.16 230.79

[32,] 53.42 0.07 792.05 230.79

[33,] -16.94 0.09 -195.31 230.79

[34,] -8.79 0.09 -93.46 230.79

[35,] -49.66 0.10 -474.25 230.79

[36,] -66.31 0.11 -602.86 230.79

[37,] -0.10 0.10 -0.91 230.79

> performance\_2

[1] 0.4525547

round(Result\_2,2)

Estimate SE Tvalue minusll

[1,] 4.66 0.10 44.60 241.32

[2,] 49.86 0.10 510.51 241.32

[3,] 44.56 0.10 464.53 241.32

[4,] -4.96 0.10 -50.56 241.32

[5,] 34.88 0.09 404.55 241.32

[6,] 10.10 0.08 120.03 241.32

[7,] 17.60 0.09 200.69 241.32

[8,] 3.52 0.09 40.76 241.32

[9,] 21.08 0.08 251.45 241.32

[10,] 41.70 0.09 460.40 241.32

[11,] 4.79 0.07 68.38 241.32

[12,] -32.05 0.07 -477.36 241.32

[13,] 1.50 0.07 22.12 241.32

[14,] 9.20 0.07 136.00 241.32

[15,] 16.25 0.09 185.60 241.32

[16,] -64.20 0.09 -679.44 241.32

[17,] -27.69 0.10 -265.09 241.32

[18,] -0.39 0.11 -3.55 241.32

[19,] -2.52 0.10 -24.12 241.32

[20,] -7.82 0.10 -80.05 241.32

[21,] 17.33 0.10 180.62 241.32

[22,] 14.33 0.10 146.02 241.32

[23,] -6.63 0.09 -76.92 241.32

[24,] 0.38 0.08 4.54 241.32

[25,] 14.40 0.09 164.18 241.32

[26,] -25.70 0.09 -297.26 241.32

[27,] -0.62 0.08 -7.40 241.32

[28,] -12.47 0.09 -137.70 241.32

[29,] 16.65 0.07 237.76 241.32

[30,] 27.04 0.07 402.79 241.32

[31,] 2.62 0.07 38.76 241.32

[32,] -7.84 0.07 -115.82 241.32

[33,] 2.10 0.09 24.03 241.32

[34,] 18.53 0.09 196.12 241.32

[35,] -9.62 0.10 -92.05 241.32

[36,] -27.16 0.11 -247.09 241.32

[37,] -0.15 0.10 -1.40 241.32

> performance\_2

[1] 0.4744526